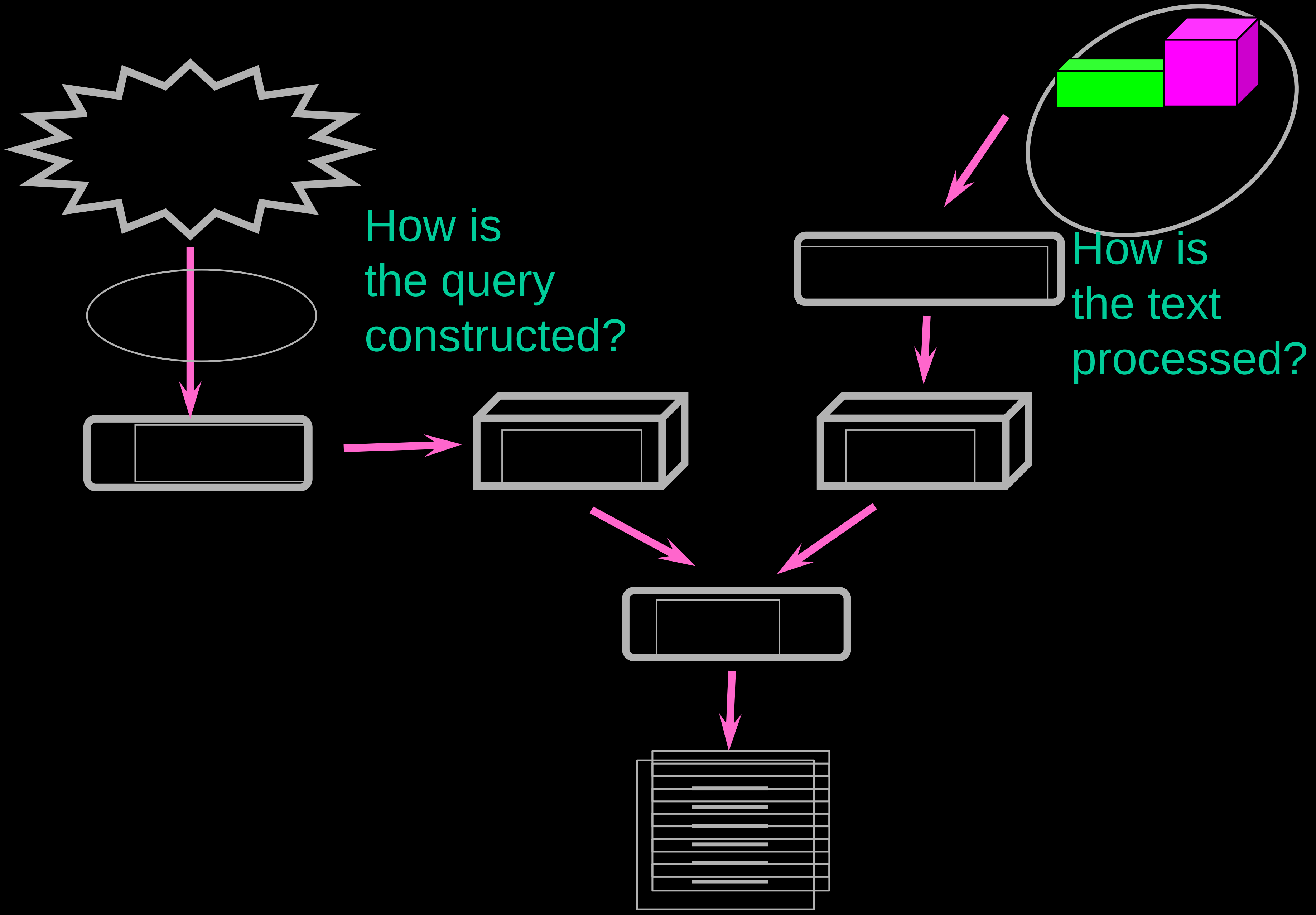
Lecture 4

IR Modeling

Traditional IR Models

- Content Analysis
  - Statistical Characteristics of Text Collections
    - Zipf distribution
    - Statistical dependence
  - Term Vector Representations
- Inverted Indexes
- Ranking


How is
the query
constructed?
How is
the text
processed?

A Taxonomy of IR Models

- Boolean Model(Set Theory)
  - Extended Boolean model
  - Fuzzy model
- Vector Model(Algebraic Theory)
  - Generalized vector model
  - Latent semantic index
  - Neural networks
- Probabilistic Model
  - Inference network
  - Belief network

Basic Concepts of Classic IR

- index terms (usually nouns): index and summarize
- weight of index terms
- Definition
  - $K=\{k_1, \ldots, k_t\}$: a set of all index terms
  - $w_{i,j}$: a weight of an index term k_i of a document d_j
  - $\vec{d_j}=(w_{1,j}, w_{2,j}, \ldots, w_{t,j})$: an *index term vector* for the document d_j
  - $g_i(\vec{d_j})= w_{i,j}$

$w_{i,j}$ associated with (k_i,d_j) tells us nothing about $w_{i+1,j}$ associated with (k_{i+1},d_j)

- assumption
  - index term weights are *mutually independent*

The terms *computer* and *network* in the area of computer networks

Boolean Model

- The index term weight variables are all binary, i.e., $w_{i,j} \in \{0,1\}$
- A query q is a Boolean expression (and, or, not)
- $\vec{q}_{dnf}$: the *disjunctive normal form* for q
- $\vec{q}_{cc}$: conjunctive components of $\vec{q}_{dnf}$
- sim(d_j,q): similarity of d_j to q
  - 1: if $\exists \vec{q}_{cc} \mid (\vec{q}_{cc} \in \vec{q}_{dnf} \wedge (\forall k_i, g_i(\vec{d}_j) = g_i(\vec{q}_{cc}))$
  - 0: otherwise

dj is relevant to q

Boolean Model (*Continued*)

- advantage: simple
- disadvantage
  - binary decision (relevant or non-relevant) without grading scale
  - exact match (no partial match)
    - e.g., $d_j=(0,1,0)$ is non-relevant to $q=(k_a \wedge (k_b \vee \neg k_c))$
  - retrieve too few or too many documents

Basic Vector Space Model

- *Term vector* representation of
  documents $D_i=(a_{i1}, a_{i2}, \ldots, a_{it})$
  queries $Q_j=(q_{j1}, q_{j2}, \ldots, q_{jt})$
- t distinct terms are used to characterize content.
- Each term is identified with a term vector T.
- t vectors are linearly independent.
- Any vector is represented as a linear combination of the t term vectors.
- The rth document D_r can be represented as a document vector, written as

$$D_r = \sum_{i=1}^{t} a_{ri} T_i$$

Document Collection

- A collection of *n* documents can be represented in the vector space model by a term-document matrix.
- An entry in the matrix corresponds to the "weight" of a term in the document; zero means the term has no significance in the document or it simply doesn't exist in the document.

$$
\begin{array}{c|cccc}
 & T_1 & T_2 & \cdots & T_t \\
\hline
D_1 & w_{11} & w_{21} & \cdots & w_{t1} \\
D_2 & w_{12} & w_{22} & \cdots & w_{t2} \\
\vdots & \vdots & \vdots & & \vdots \\
\vdots & \vdots & \vdots & & \vdots \\
D_n & w_{1n} & w_{2n} & \cdots & w_{tn}
\end{array}
$$

Documents as vectors

- So we have a |V|-dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- Very high-dimensional: hundreds of millions of dimensions when you apply this to a web search engine
- This is a very sparse vector - most entries are zero. $d_j = (w_{1j}, w_{2j}, \ldots, w_{tj})$

Queries as vectors

- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- proximity ≈ inverse of distance
- Recall: We do this because we want to get away from the you're-either-in-or-out Boolean model.
- Instead: rank **more relevant** documents higher than less relevant documents

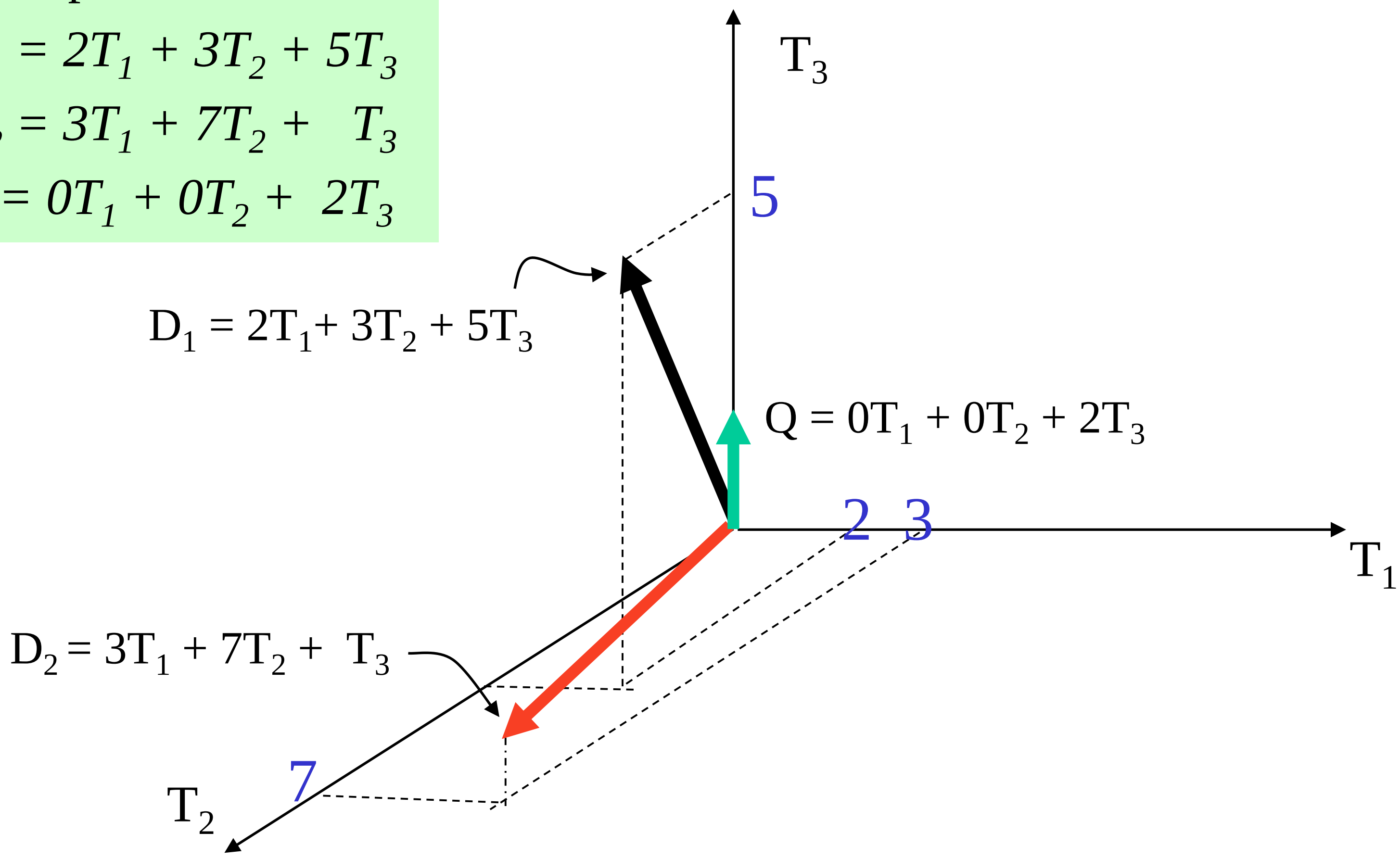
Graphic Representation

Example:

$D_1 = 2T_1 + 3T_2 + 5T_3$

$D_2 = 3T_1 + 7T_2 + T_3$

$Q = 0T_1 + 0T_2 + 2T_3$



Rank Retrieval over Vector Space

- Retrieval based on *similarity* between query and documents.
- Output documents are ranked according to similarity to query.

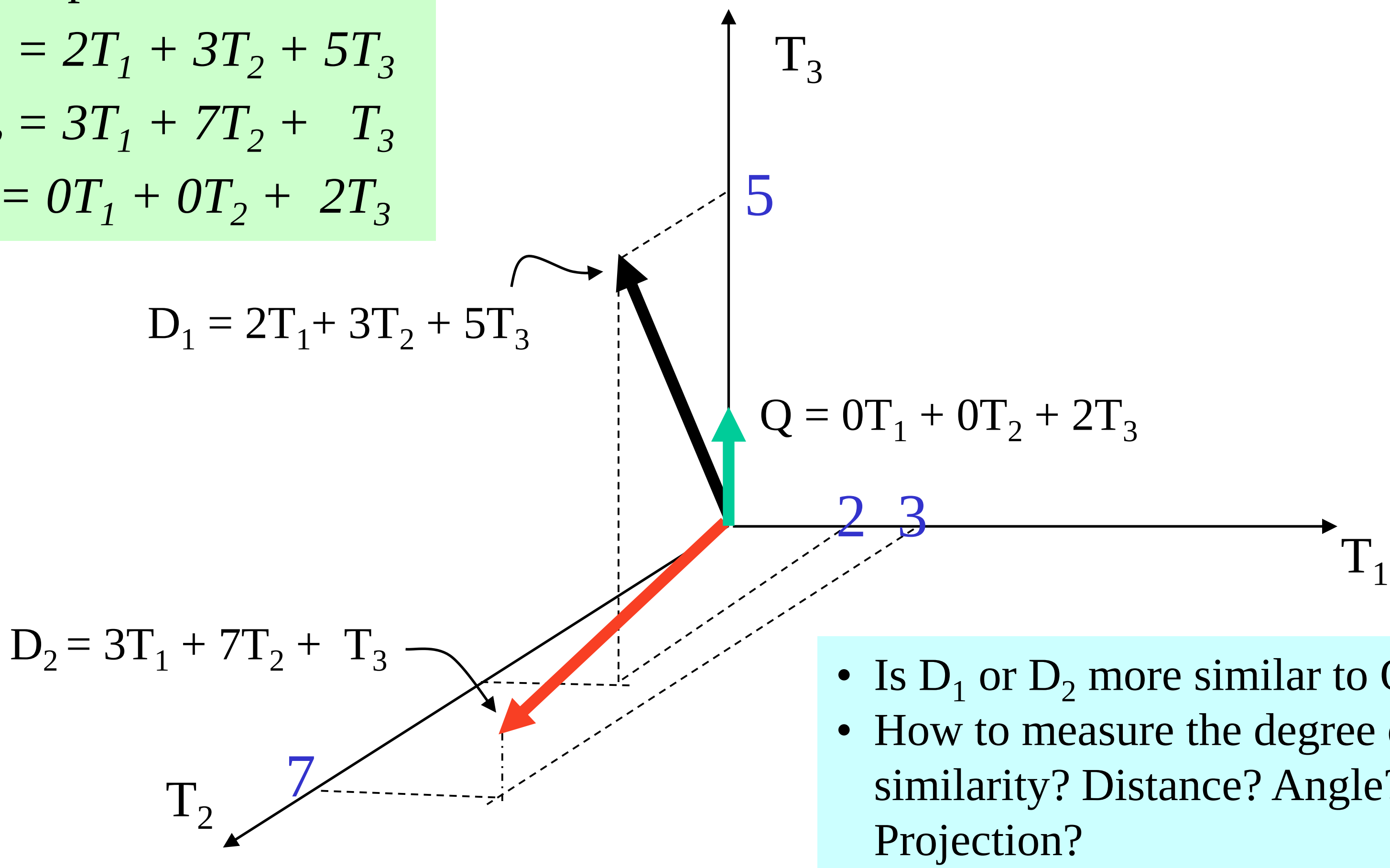
Graphic Representation

Example:

$D_1 = 2T_1 + 3T_2 + 5T_3$

$D_2 = 3T_1 + 7T_2 + T_3$

$Q = 0T_1 + 0T_2 + 2T_3$



- Is D_1 or D_2 more similar to Q?
- How to measure the degree of similarity? Distance? Angle? Projection?

Similarity Measure

- A similarity measure is a function that computes the *degree of similarity* between two vectors.

- Using a similarity measure between the query and each document:
  - It is possible to rank the retrieved documents in the order of presumed relevance.
  - It is possible to enforce a certain threshold so that the size of the retrieved set can be controlled.

Similarity Measure

- measure by product of two vectors

  $$x \bullet y = |x|\,|y| \cos\alpha$$

- document-query similarity

document vector:

$$D_r = \sum_{i=1}^{t} a_{ri} T_i$$

term vector:

$$Q_s = \sum_{j=1}^{t} q_{sj} T_j$$

$$D_r \bullet Q_s = \sum_{i,j=1}^{t} a_{ri} q_{sj} T_i \bullet T_j$$

- how to determine the vector components and term correlations?

Similarity Measure (*Continued*)

- vector components

$$A = \begin{array}{c} \\ D_1 \\ D_2 \\ \vdots \\ D_n \end{array} \begin{array}{c} \begin{array}{cccc} T_1 & T_2 & T_3 & T_t \end{array} \\ \left| \begin{array}{cccc} a_{11} & a_{12} & \cdots & a_{1t} \\ a_{21} & a_{22} & \cdots & a_{2t} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nt} \end{array} \right| \end{array}$$

Similarity Measure (*Continued*)

- term correlations $T_i \bullet T_j$ are not available assumption: term vectors are orthogonal

$$T_i \bullet T_j = 0 \ (i \neq j) \quad T_i \bullet T_j = 1 \ (i = j)$$

- Assume that terms are uncorrelated.

$$sim(D_r, Q_s) = \sum_{i,j=1}^{t} a_{ri} q_{sj}$$

- Similarity measurement between documents

$$sim(D_r, D_s) = \sum_{i,j=1}^{t} a_{ri} a_{sj}$$

Simple query-document similarity computation

- $D_1=2T_1+3T_2+5T_3$ $D_2=3T_1+7T_2+1T_3$
  $Q=0T_1+0T_2+2T_3$
- similarity computations for uncorrelated terms
  $sim(D_1,Q)=2\bullet 0+3\bullet 0+5\bullet 2=10$
  $sim(D_2,Q)=3\bullet 0+7\bullet 0+1\bullet 2=2$
- D_1 is preferred

Simple query-document similarity computation (*Continued*)

-

	T_1	T_2	T_3
T_1	1	0.5	0
T_2	0.5	1	-0.2
T_3	0	-0.2	1

- similarity computations for correlated terms

$$sim(D_1,Q)=(2T_1+3T_2+5T_3) \bullet (0T_1+0T_2+2T_3)$$
$$=4T_1 \bullet T_3+6T_2 \bullet T_3+10T_3 \bullet T_3$$
$$=-6*0.2+10*1=8.8$$

$$sim(D_2,Q)=(3T_1+7T_2+1T_3) \bullet (0T_1+0T_2+2T_3)$$
$$=6T_1 \bullet T_3+14T_2 \bullet T_3+2T_3 \bullet T_3$$
$$=-14*0.2+2*1=-0.8$$

- D_1 is preferred

Vector Model

- $w_{i,j}$: a positive, *non-binary weight* for (k_i,d_j)
- $w_{i,q}$: a positive, *non-binary weight* for (k_i,q)
- $\vec{q}=(w_{1,q}, w_{2,q}, \ldots, w_{t,q})$: a query vector, where t is the total number of index terms in the system
- $\vec{d}_j= (w_{1,j}, w_{2,j}, \ldots, w_{t,j})$: a document vector

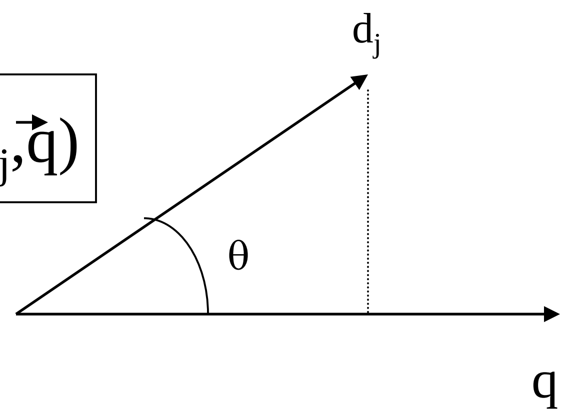
Similarity of document d_j w.r.t. query q

- The correlation between vectors $\vec{d}_j$ and $\vec{q}$

$$sim(d_j, q) = \frac{\vec{d}_j \bullet \vec{q}}{|\vec{d}_j| \times |\vec{q}|}$$

$$= \frac{\sum_{i=1}^{t} w_{i,j} \times w_{i,q}}{\sqrt{\sum_{i=1}^{t} w_{i,j}^2} \times \sqrt{\sum_{j=1}^{t} w_{i,q}^2}}$$

$\boxed{\cos(\vec{d}_j, \vec{q})}$



- $|\vec{q}|$ does not affect the ranking
- $|\vec{d}_j|$ provides a normalization

document ranking

- Similarity (*i.e.*, sim(q, d_j)) between 0 and 1.
- Retrieve the documents with a degree of similarity above a predefined threshold (allow partial matching)

Term weighting techniques

- IR problem vs Classification problem:
  - user query: a specification of a set A of objects
  - classification problem: determine which documents are in the set A (*relevant*), which ones are not (*non-relevant*)
  - intra-cluster similarity
    - the features better describe the objects in the set A
    - tf factor in vector model
      the raw frequency of a term k_i inside a document d_j
  - inter-cluster similarity
    - the features better distinguish the the objects in the set A from the remaining objects in the collection C
    - idf factor (inverse document frequency) in vector model
      the inverse of the frequency of a term k_i among the documents in the collection

Definition of *tf*

- N: total number of documents in the system
- n_i: the number of documents in which the index term k_i appears
- $freq_{i,j}$: the raw frequency of term k_i in the document d_j
- $f_{i,j}$: the *normalized frequency* of term k_i in document d_j

$$f_{i,j} = \frac{freq_{i,j}}{\max_l \, freq_{l,j}} \qquad (0\sim1)$$

Term t_l has maximum frequency in the document d_j

Definition of *idf* and *tf-idf* scheme

- idf_i: inverse document frequency for k_i

$$idf_i = \log \frac{N}{n_i}$$

- $w_{i,j}$: term-weighting by *tf-idf* scheme

$$w_{i,j} = f_{i,j} \times \log \frac{N}{n_i}$$

- *query term* weight (Salton and Buckley)

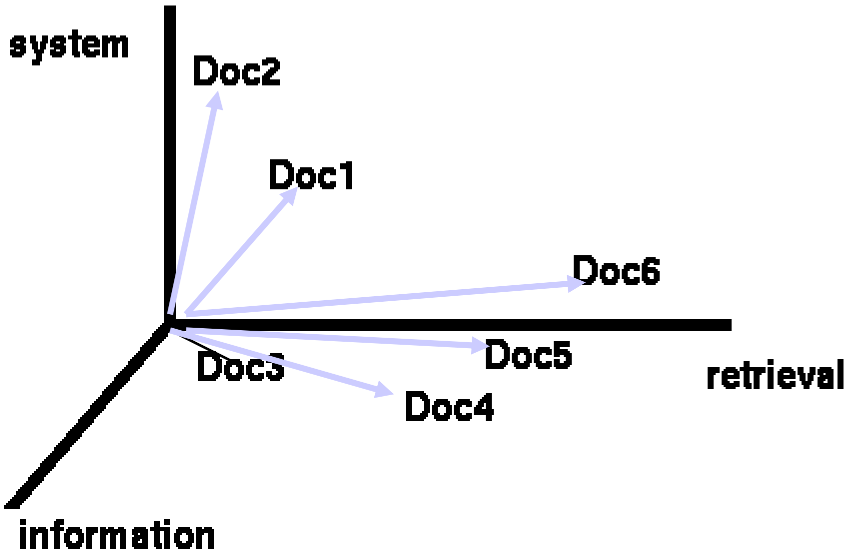
$$w_{i,q} = (0.5 + \frac{0.5\, freq_{i,q}}{\max_l \ freq_{i,q}}) \times \log \frac{N}{n_i}$$

$freq_{i,q}$: the raw frequency of the term k_i in q

Analysis of vector model

- advantages
  - its *term-weighting* scheme improves *retrieval performance*
  - its *partial matching* strategy allows retrieval of documents that *approximate* the query conditions
  - its *cosine ranking* formula sorts the documents according to their *degree of similarity* to the query
- disadvantages
  - indexed terms are assumed to be *mutually independently*

Documents in 3D Space

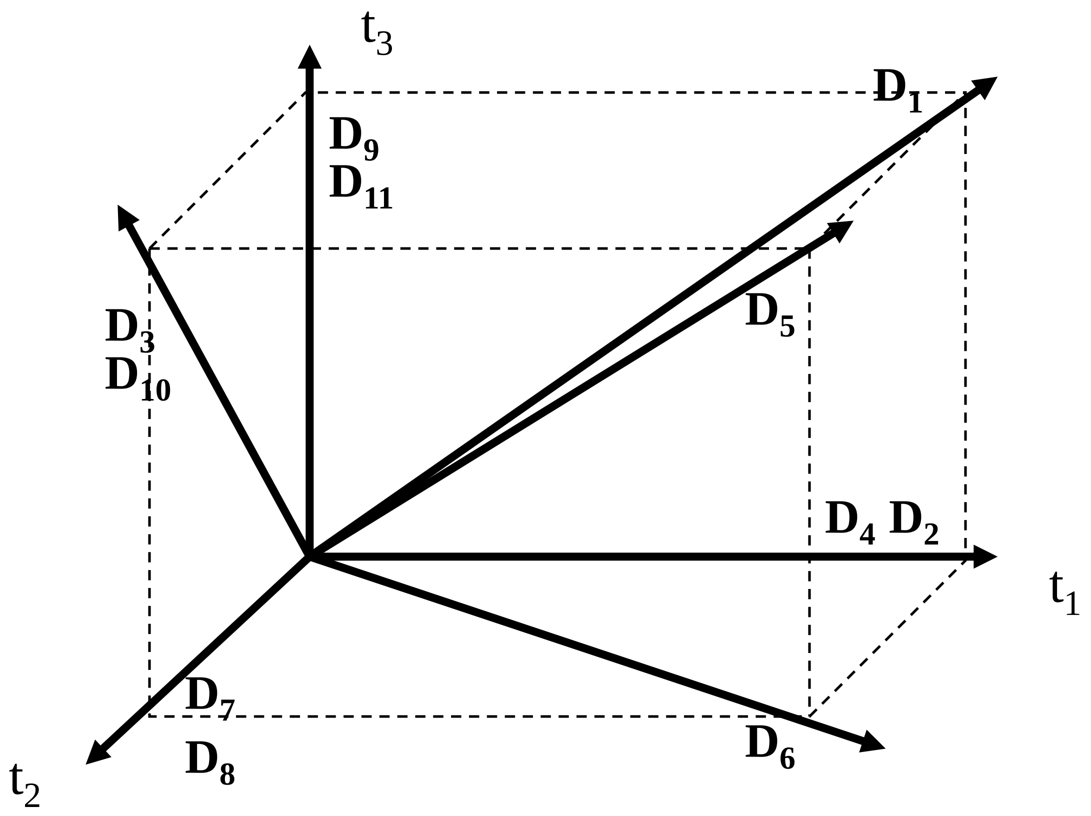


Assumption: Documents that are “close together” in space are similar in meaning.

Vector Space Model

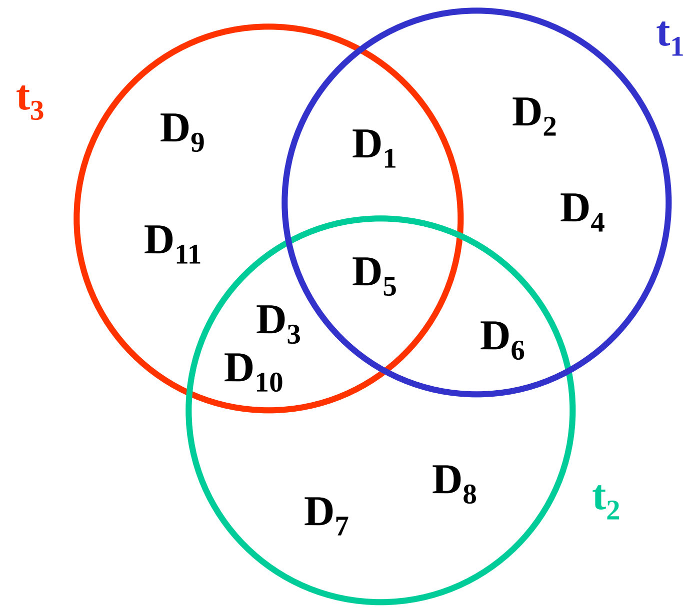
- Documents are represented as vectors in term space
  - Terms are usually stems
  - Documents represented by binary vectors of terms
- Queries represented the same as documents
- Query and Document weights are based on length and direction of their vector
- A vector distance measure between the query and documents is used to rank retrieved documents

Documents in Vector Space



Vector Space Documents and Queries

docs	*t1*	*t2*	*t3*	RSV=Q.Di
D1	1	0	1	4
D2	1	0	0	1
D3	0	1	1	5
D4	1	0	0	1
D5	1	1	1	6
D6	1	1	0	3
D7	0	1	0	2
D8	0	1	0	2
D9	0	0	1	3
D10	0	1	1	5
D11	0	0	1	3
Q	1 *q1*	2 *q2*	3 *q3*	



Similarity Measures

$$|Q \cap D|$$

Simple matching (coordination level match)

$$2\frac{|Q \cap D|}{|Q| + |D|}$$

Dice's Coefficient

$$\frac{|Q \cap D|}{|Q \cup D|}$$

Jaccard's Coefficient

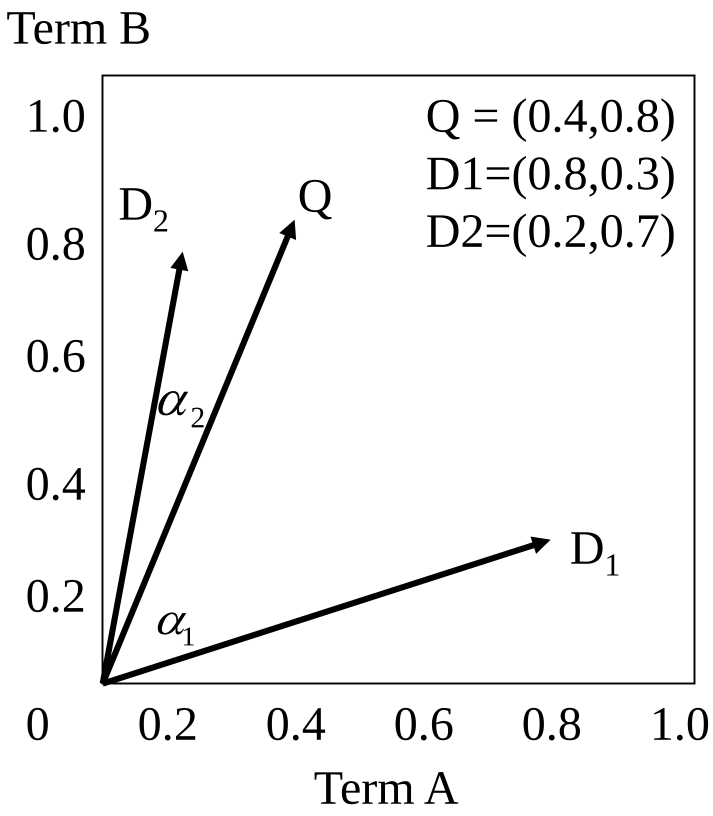
$$\frac{|Q \cap D|}{|Q|^{1/2} \times |D|^{1/2}}$$

Cosine Coefficient

$$\frac{|Q \cap D|}{\min(|Q|, |D|)}$$

Overlap Coefficient

Vector Space with Term Weights and Cosine Matching



$D_i=(d_{i1},w_{di1};d_{i2}, w_{di2};\ldots;d_{it}, w_{dit})$
$Q =(q_{i1},w_{qi1};q_{i2}, w_{qi2};\ldots;q_{it}, w_{qit})$

$$sim(Q,D_i) = \frac{\sum_{j=1}^{t} w_{q_j} w_{d_{ij}}}{\sqrt{\sum_{j=1}^{t}(w_{q_j})^2 \sum_{j=1}^{t}(w_{d_{ij}})^2}}$$

$$sim(Q,D2) = \frac{(0.4\cdot 0.2)+(0.8\cdot 0.7)}{\sqrt{[(0.4)^2+(0.8)^2]\cdot[(0.2)^2+(0.7)^2]}}$$

$$= \frac{0.64}{\sqrt{0.42}} = 0.98$$

$$sim(Q,D_1) = \frac{.56}{\sqrt{0.58}} = 0.74$$

Probabilistic Model

- Given a query, there is an *ideal answer set*
  - a set of documents which contains exactly the relevant documents and no other
- query process
  - a process of specifying *the properties* of an ideal answer set
- problem: what are the properties?

Probabilistic Principle

- Given a *user query* q and a *document* d_j in the collection, the probabilistic model estimates the probability that user will find d_j relevant
- assumptions
  - The probability of relevance depends on query and document representations only
  - There is a subset of all documents which the user prefers as the answer set for the query q
- Given a query, the probabilistic model assigns to each document dj a measure of its similarity to the query

$$\frac{P(d_j \ relevant-to\, q)}{P(d_j \ nonrelevant-to\, q)}$$

Probabilistic Principle

- $w_{i,j} \in \{0,1\}$, $w_{i,q} \in \{0,1\}$: the index term weight variables are all binary non-relevant
- q: a query which is a subset of index terms
- R: the set of documents known to be *relevant*
- $\overline{R}$: the set of documents known to be *non-relevant*
- $P(R|\vec{d}_j)$: the probability that the document d_j is *relevant* to the query q
- $P(\overline{R}|\vec{dj})$: the probability that d_j is *non-relevant* to q

Similarity

- sim(d_j,q): the similarity of the document d_j to the query q

$$sim(d_j,q) = \frac{P(R \mid \overrightarrow{d_j})}{P(\overline{R} \mid \overrightarrow{d_j})}$$

(by definition)

$$sim(d_j,q) = \frac{P(\overrightarrow{d_j} \mid R) \times P(R)}{P(\overrightarrow{d_j} \mid \overline{R}) \times P(\overline{R})}$$

(Bayes' rule)

$$sim(d_j,q) \approx \frac{P(\overrightarrow{d_j} \mid R)}{P(\overrightarrow{d_j} \mid \overline{R})}$$

(P(R) and P($\overline{R}$) are the same for all documents)

$P(\vec{d}_j \mid R)$: the probability of randomly selecting the document d_j from the set of R of relevant documents

$P(R)$: the probability that a document randomly selected from the entire collection is relevant

$$sim(d_j, q) \approx \frac{P(\overrightarrow{d_j} \mid R)}{P(\overrightarrow{d_j} \mid \overline{R})}$$

$$= \log \frac{\prod_{i=1}^{t} (P(k_i \mid R))^{g_i(\overrightarrow{d_j})} \times (P(\overline{k}_i \mid R))^{1-g_i(\overrightarrow{d_j})}}{\prod_{i=1}^{t} (P(k_i \mid \overline{R}))^{g_i(\overrightarrow{d_j})} \times (P(\overline{k}_i \mid \overline{R}))^{1-g_i(\overrightarrow{d_j})}}$$

$P(k_i|R)$: the probability that the index term k_i is present in a document randomly selected from the set R.

$P(\overline{k}_i|R)$: the probability that the index term k_i is not present in a document randomly selected from the set R.

independence assumption of index terms

$$= \sum_{i=1}^{t} \log \frac{(P(k_i \mid R))^{g_i(\overrightarrow{d_j})} \times (P(\overline{k}_i \mid R))^{1-g_i(\overrightarrow{d_j})}}{(P(k_i \mid \overline{R}))^{g_i(\overrightarrow{d_j})} \times (P(\overline{k}_i \mid \overline{R}))^{1-g_i(\overrightarrow{d_j})}}$$

$$= \sum_{i=1}^{t} \log \frac{(P(k_i \mid R) \times P(\overline{k}_i \mid \overline{R}))^{g_i(\overrightarrow{d_j})} \times (P(\overline{k}_i \mid R))}{(P(k_i \mid \overline{R}) \times P(\overline{k}_i \mid R))^{g_i(\overrightarrow{d_j})} \times (P(\overline{k}_i \mid \overline{R}))}$$

$$= \sum_{i=1}^{t} g_i(\overrightarrow{d_j}) \times \log \frac{P(k_i \mid R) \times P(\overline{k}_i \mid \overline{R})}{P(k_i \mid \overline{R}) \times P(\overline{k}_i \mid R)} + \sum_{i=1}^{t} \frac{P(\overline{k}_i \mid R)}{P(\overline{k}_i \mid \overline{R})}$$

$$= \sum_{i=1}^{t} g_i(\overrightarrow{d_j}) \times \log \frac{P(k_i \mid R) \times (1 - P(k_i \mid \overline{R}))}{P(k_i \mid \overline{R}) \times (1 - P(k_i \mid R))} + \sum_{i=1}^{t} \frac{P(\overline{k}_i \mid R)}{P(\overline{k}_i \mid \overline{R})}$$

$$sim(d_j, q) \approx \frac{P(\vec{d_j} \mid R)}{P(\vec{d_j} \mid \overline{R})}$$

$$= \sum_{i=1}^{t} g_i(\vec{d_j}) \times \log \frac{P(k_i \mid R) \times (1 - P(k_i \mid \overline{R}))}{P(k_i \mid \overline{R}) \times (1 - P(k_i \mid R))} + \sum_{i=1}^{t} \frac{P(\overline{k}_i \mid R)}{P(\overline{k}_i \mid \overline{R})}$$

$$= \sum_{i=1}^{t} g_i(\vec{d_j}) \times (\log \frac{P(k_i \mid R)}{(1 - P(k_i \mid R))}) + \log \frac{(1 - P(k_i \mid \overline{R}))}{P(k_i \mid \overline{R})}) + \sum_{i=1}^{t} \frac{P(\overline{k}_i \mid R)}{P(\overline{k}_i \mid \overline{R})}$$

$$\approx \sum_{i=1}^{t} g_i(\vec{d_j}) \times (\log \frac{P(k_i \mid R)}{(1 - P(k_i \mid R))}) + \log \frac{(1 - P(k_i \mid \overline{R}))}{P(k_i \mid \overline{R})})$$

Problem: where is the set R?

Initial guess

- $P(k_i|R)$ is constant for all index terms k_i.

$$p(k_i \mid R) = 0.5$$

- The distribution of index terms among the non-relevant documents can be approximated by the distribution of index terms among all the documents in the collection.

$$P(k_i \mid \overline{R}) = \frac{n_i}{N}$$

(assume $N >> |R|$, $N \approx |\overline{R}|$)

Initial ranking

- V: a subset of the documents initially retrieved and ranked by the probabilistic model (*top r documents*)
- V_i: subset of V composed of documents which contain the index term k_i
- Approximate $P(k_i|R)$ by the distribution of the index term k_i among the documents retrieved so far.

$$P(k_i \mid R) = \frac{V_i}{V}$$

- Approximate $P(k_i|\overline{R})$ by considering that all the non-retrieved documents are not relevant.

$$P(k_i \mid \overline{R}) = \frac{n_i - V_i}{N - V}$$

Small values of V and V_i

$$P(k_i \mid R) = \frac{V_i}{V}$$

$$P(k_i \mid \overline{R}) = \frac{n_i - V_i}{N - V}$$

a problem when V=1 and V_i=0

- alternative 1

$$P(k_i \mid R) = \frac{V_i + 0.5}{V + 1}$$

$$P(k_i \mid \overline{R}) = \frac{n_i - V_i + 0.5}{N - V + 1}$$

- alternative 2

$$P(k_i \mid R) = \frac{V_i + \frac{n_i}{N}}{V + 1}$$

$$P(k_i \mid \overline{R}) = \frac{n_i - V_i + \frac{n_i}{N}}{N - V + 1}$$

Analysis of Probabilistic Model

- advantage
  - documents are ranked in decreasing order of their probability of being relevant
- disadvantages
  - the need to guess the initial separation of documents into relevant and non-relevant sets
  - do not consider the frequency with which an index terms occurs inside a document
  - the independence assumption for index terms

Comparison of classic models

- Boolean model: the weakest classic model
- Vector model is expected to outperform the probabilistic model with general collections (Salton and Buckley)

Alternative Set Theoretic Models -Fuzzy Set Model

- Model
  - a query term: a fuzzy set
  - a document: degree of membership in this set
  - membership function
    - Associate membership function with the elements of the class
    - 0: no membership in the set
    - 1: fully membership
    - 0~1: marginal elements of the set

documents

Fuzzy Set Theory

a class

- A fuzzy subset A of a universe of discourse U is characterized by a membership function μ_A: U→[0,1] which associates with each element u of U a number $\mu_A(u)$ in the interval [0,1]

a document

  - complement: $\mu_{\overline{A}}(u) = 1 - \mu_A(u)$
  - union: $\mu_{A \cup B}(u) = \max(\mu_A(u), \mu_B(u))$
  - intersection: $\mu_{A \cap B}(u) = \min(\mu_A(u), \mu_B(u))$

Examples

- Assume $U=\{k_1, k_2, k_3, k_4, k_5, k_6\}$
- Let A and B be $\{k_1, k_2, k_3\}$ and $\{k_2, k_3, k_4\}$, respectively.
- Assume $\mu_A=\{k_1/0.8, k_2/0.7, k_3/0.6, k_4/0, k_5/0, k_6/0\}$ and $\mu_B=\{k_1/0, k_2/0.6, k_3/0.8, k_4/0.9, k_5/0, k_6/0\}$
- $\mu_{\overline{A}}(u) = 1 - \mu_A(u) = \{k_1{:}0.2, k_2{:}0.3, k_3{:}0.4, k_4{:}1, k_5{:}1, k_6{:}1\}$
- $\mu_{A\cup B}(u) = \max(\mu_A(u), \mu_B(u)) = \{k_1{:}0.8, k_2{:}0.7, k_3{:}0.8, k_4{:}9, k_5{:}0, k_6{:}0\}$
- $\mu_{A\cap B}(u) = \min(\mu_A(u), \mu_B(u)) = \{k_1{:}0, k_2{:}0.6, k_3{:}0.6, k_4{:}0, k_5{:}0, k_6{:}0\}$

Fuzzy Information Retrieval

- basic idea
  - Expand the set of index terms in the query with related terms (from the thesaurus) such that additional relevant documents can be retrieved
  - A thesaurus can be constructed by defining a term-term correlation matrix $\vec{c}$ whose rows and columns are associated to the index terms in the document collection

keyword correlation matrix

Fuzzy Information Retrieval

(Continued)

- normalized correlation factor $c_{i,l}$ between two terms k_i and k_l (0~1)

$$c_{i,l} = \frac{n_{i,l}}{n_i + n_l - n_{i,l}} \quad \text{where} \begin{cases} n_i \text{ is \# of documents containing term } k_i \\ n_l \text{ is \# of documents containing term } k_l \\ n_{i,l} \text{ is \# of documents containing } k_i \text{ and } k_l \end{cases}$$

- In the fuzzy set associated to each index term k_i, a document d_j has a degree of membership $\mu_{i,j}$

$$\mu_{i,j} = 1 - \prod_{k_l \in d_j} (1 - c_{i,l})$$

Example

Query $q = k_a \wedge (k_b \vee \neg k_c)$

disjunctive normal form $\vec{q}_{dnf} = (1,1,1) \vee (1,1,0) \vee (1,0,0)$

(1) the degree of membership in a disjunctive fuzzy set is computed using an algebraic sum *(instead of max function)* *more smoothly*
(2) the degree of membership in a conjunctive fuzzy set is computed using an algebraic product (*instead of min function*)

$$\mu_{q,j} = \mu_{cc1+cc2+cc3,j}$$
$$= 1 - \prod_{i=1}^{3} (1 - \mu_{cc_i,j})$$
$$= 1 - (1 - \mu_{a,j}\mu_{b,j}\mu_{c,j}) \times (1 - \mu_{a,j}\mu_{b,j}(1 - \mu_{c,j})) \times (1 - \mu_{a,j}(1 - \mu_{b,j})(1 - \mu_{c,j}))$$

Recall $\mu_{\overline{A}}(u) = 1 - \mu_A(u)$

Fuzzy Set Model Summary

- Matching degree (the matching of a “document” to the “query”)
- Membership μ_A

$$\text{OR: } \mu_{A \cup B} = \max(\mu_A, \mu_B)$$

$$\text{AND: } \mu_{A \wedge B} = \min(\mu_A, \mu_B)$$

$$\text{NOT: } \mu_{\bar{A}} = 1 - \mu_A$$

- Binary decision→grade membership
- term-term correlation
- Fuzzy Set operators (min/max --> fuzzy operator)
- “index-term” set ⟷ “related-term”

  (query) ↑

  “thesaurus” 同義字字典

Extended Boolean Model

- (different from Boolean) Partial matching & terms weighting
- Boolean query $_{\text{e.g.}}$ $q = K_1 \Lambda K_2$

  $D_1 \supset K_1$, $D_2 \supset K_2$, $D_3 \supset K_3$, K_4 query results are the same
- Using "Similarity" to represent "matching degree"

  $\Rightarrow$ *Sim*(q,dj)

  example: distance,algebraic mean,...

Alternative Algebraic Model: Generalized Vector Space Model

- independence of index terms
  - $\vec{k_i}$: a vector associated with the index term k_i
  - the set of vectors $\{\vec{k_1}, \vec{k_2}, \ldots, \vec{k_t}\}$ is linearly independent
    - orthogonal: $\vec{k}_i \bullet \vec{k}_j = 0$ for $i \neq j$
  - The index term vectors are assumed linearly independent but are not pairwise orthogonal in generalized vector space model
  - The index term vectors, which are not seen as the basis of the space, are composed of *smaller components* derived from the particular collection.

Generalized Vector Space Model

- $\{k_1, k_2, \ldots, k_t\}$: index terms in a collection
- $w_{i,j}$: binary weights associated with the term-document pair $\{k_i, d_j\}$
- The patterns of term *co-occurrence* (inside documents) can be represented by a set of 2^t *minterms*

$m_1=(0, 0, \ldots, 0)$: point to documents containing none of index terms
$m_2=(1, 0, \ldots, 0)$: point to documents containing the index term k_1 only
$m_3=(0,1,\ldots,0)$: point to documents containing the index term k_2 only
$m_4=(1,1,\ldots,0)$: point to documents containing the index terms k_1 and k_2

…

$m_{2^t}=(1, 1, \ldots, 1)$: point to documents containing all the index terms

- $g_i(m_j)$: return the weight $\{0,1\}$ of the index term k_i in the minterm m_j $(1 \le i \le t)$

Generalized Vector Space Model

(*Continued*)

$$\vec{m}_1 = (1,0,\ldots,0,0)$$

$$\vec{m}_2 = (0,1,\ldots,0,0)$$

$$\ldots$$

$$\vec{m}_{2^t} = (0,0,\ldots,0,1)$$

$$\vec{m}_i \bullet \vec{m}_j = 0 \;\; for \; i \neq j$$

(the set of $\vec{m}_i$ are pairwise orthogonal)

- $\vec{m}_i$ (2^t-tuple vector) is associated with minterm m_i (t-tuple vector)
- e.g., $\vec{m}_4$ is associated with m_4 containing k_1 and k_2, and no others
- co-occurrence of index terms inside documents: dependencies among index terms

Generalized Vector Space Model

(*Continued*)

- Determine the index vector $\vec{k_i}$ associated with the index term k_i

$$\vec{k}_i = \frac{\sum_{\forall r, g_i(m_r)=1} c_{i,r} \vec{m}_r}{\sqrt{\sum_{\forall r, g_i(m_r)=1} c_{i,r}^2}}$$

Collect all the vectors $\vec{m}_r$ in which the index term k_i is in state 1.

$$c_{i,r} = \sum_{d_j | g_l(\vec{d}_j) = g_l(m_r) \ for\ all\ l} w_{i,j}$$

Sum up $w_{i,j}$ associated with the index term k_i and document d_j whose term occurrence pattern coincides with minterm m_r

Generalized Vector Space Model

(*Continued*)

- $\overrightarrow{k_i} \bullet \overrightarrow{k_j}$ quantifies a degree of correlation between k_i and k_j

$$\vec{k}_i \bullet \vec{k}_j = \sum_{\forall r | g_i(m_r)=1 \wedge g_j(m_r)=1} c_{i,r} \times c_{j,r}$$

- standard cosine similarity is adopted

$$\vec{d}_j = \sum_{\forall i} w_{i,j} \vec{k}_i \qquad \vec{q} = \sum_{\forall i} w_{i,q} \vec{k}_i$$

$$\vec{k}_i = \frac{\sum_{\forall r, g_i(m_r)=1} c_{i,r} \vec{m}_r}{\sqrt{\sum_{\forall r, g_i(m_r)=1} c_{i,r}^2}}$$

Latent Semantic Indexing Model

- representation of documents and queries by index terms
  - problem 1: many unrelated documents might be included in the answer set
  - problem 2: relevant documents which are not indexed by any of the query keywords are not retrieved
- possible solution: concept matching instead of index term matching
  - application in cross-language information retrieval

Basic idea

- Map each document and query vector into a lower dimensional space which is associated with concepts
- Retrieval in the reduced space may be superior to retrieval in the space of index terms

Definition

- t: the number of index terms in the collection
- N: the total number of documents
- $\vec{M}=(M_{ij})$: a term-document association matrix with t rows and N columns
- M_{ij}: a weight $w_{i,j}$ associated with the term-document pair $[k_i, d_j]$ (e.g., using tf-idf)

Singular Value Decomposition

$A \in R^{n \times n}$

(1) $A = A^T$

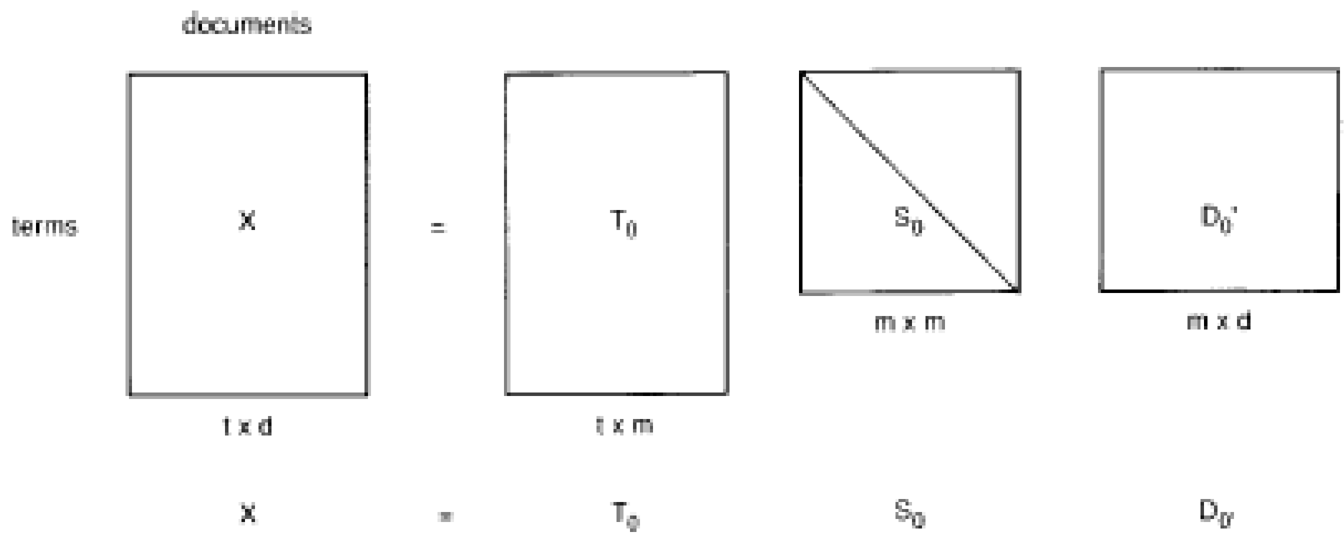
$\exists Q \in R^{n \times n} \quad st\; QQ^T = I \quad \{Q^T Q = I\}$ orthogonal

$\sin gular\;\; value\; decomposit\,ion:$

$A = QDQ^T \quad \{A^T = (QDQ^T)^T = (Q^T)^T D^T Q^T = QDQ^T = A\}$

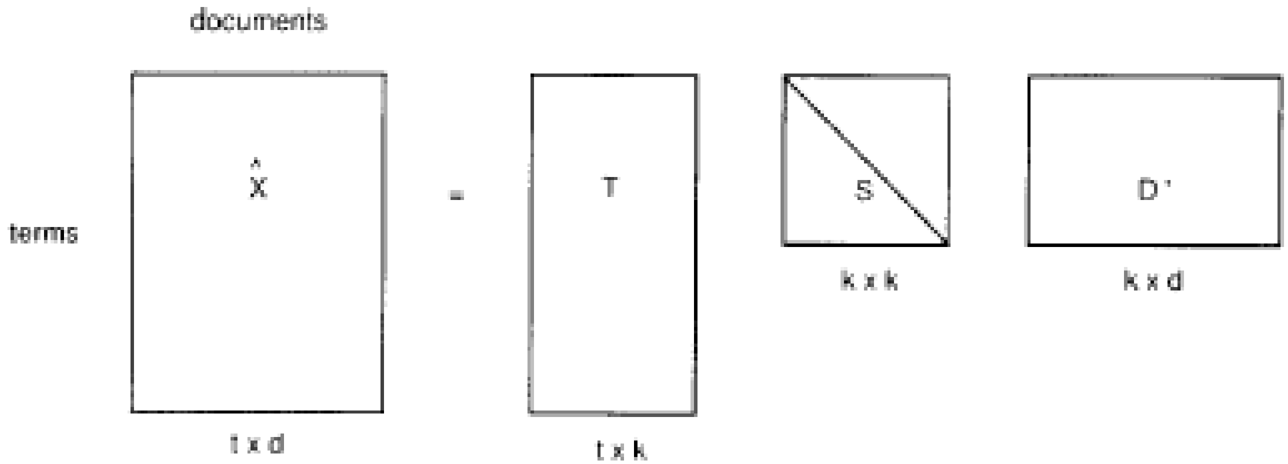
$$\text{where D} = \begin{pmatrix} \lambda_1 & & & & 0 \\ & \lambda_2 & & & \\ & & \cdot & & \\ & 0 & & \cdot & \\ & & & \cdot & \lambda_n \end{pmatrix} \quad \text{diagonal matrix}$$

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \geq 0$$



Singular value decomposition of the term x document matrix, X. Where:

T_0 has orthogonal, unit-length columns ($T_0' T_0 = I$)
D_0 has orthogonal, unit-length columns ($D_0' D_0 = I$)
S_0 is the diagonal matrix of singular values
t is the number of rows of X
d is the number of columns of X
m is the rank of X ($\leq \min(t,d)$)



$$\hat{X} = T \quad S \quad D'$$

Reduced singular value decomposition of the term x document matrix, X. Where:

T has orthogonal, unit-length columns ($T' T = I$)
D has orthogonal, unit-length columns ($D' D = I$)
S is the diagonal matrix of singular values

t is the number of rows of X
d is the number of columns of X
m is the rank of X ($\leq \min(t,d)$)
k is the chosen number of dimensions in the reduced model ($k \leq m$)

Technical Memo Example

Titles

c1:	*Human* machine *interface* for Lab ABC *computer* applications
c2:	A *survey of user* opinion of *computer system response time*
c3:	The *EPS user interface* management *system*
c4:	*System* and *human system* engineering testing of *EPS*
c5:	Relation of *user*-perceived *response time* to error measurement
m1:	The generation of random, binary, unordered *trees*
m2:	The intersection *graph* of paths in *trees*
m3:	*Graph minors* IV: Widths of *trees* and well-quasi-ordering
m4:	*Graph minors: A survey*

Terms	Documents								
	c1	c2	c3	c4	c5	m1	m2	m3	m4
human	1	0	0	1	0	0	0	0	0
interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
user	0	1	1	0	1	0	0	0	0
system	0	1	1	2	0	0	0	0	0
response	0	1	0	0	1	0	0	0	0
time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
survey	0	1	0	0	0	0	0	0	1
trees	0	0	0	0	0	1	1	1	0
graph	0	0	0	0	0	0	1	1	1
minors	0	0	0	0	0	0	0	1	1

$A \in R^{n \times n}$

(2) $A \neq A^T$

$\exists U, V \in R^{n \times n} \quad st\ U^T U = I, V^T V = I$ orthogonal

$\sin gular\ value\ decomposit\,ion:$

$(AB)^T = B^T A^T$

$A = UDV^T$

$AA^T = (UDV^T)(UDV^T)^T = (UDV^T)(VDU^T) = UD^2U^T$

$$\text{where } D = \begin{pmatrix} \lambda_1 & & & & 0 \\ & \lambda_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ 0 & & & & \cdot \quad \lambda_n \end{pmatrix}$$ diagonal matrix

$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \geq 0$

$$A = QDQ^T$$

$$AQ = QDQ^T Q = QD$$

$$where\ Q = [q_1 \quad q_2 \quad \dots \quad q_n], \quad q_i : a\ column\ vector$$

$$A[q_1 \quad q_2 \dots q_n] = [q_1 \quad q_2 \dots q_n] \begin{pmatrix} \lambda_1 & & & & & \\ & \lambda_2 & & & 0 & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & \lambda_n \end{pmatrix}$$

$$[Aq_1\ Aq_2 \dots Aq_n] = [\lambda_1 q_1 \quad \lambda_2 q_2 \quad \dots \lambda_n q_n]$$

$$Aq_1 = \lambda_1 q_1 \quad Aq_2 = \lambda_2 q_2 \quad \dots \quad Aq_n = \lambda_n q_n$$

$\lambda_1, \lambda_2, \dots, \lambda_n$ are eigenvalues of A，

Q_k is λ_k's corresponding eigenvector

Singular Value Decomposition

$\overrightarrow{M}$: $a\ term-document\ matrix\ with\ t\ rows\ and\ N\ columns$

$\overrightarrow{M} = \overrightarrow{K}\overrightarrow{S}\overrightarrow{D}^t$

$\overrightarrow{M}^t\overrightarrow{M}$: $a\ N \times N\ document-to-document\ matrix$

$\overrightarrow{M}\overrightarrow{M}^t$: $a\ t \times t\ term-to-term\ matrix$

According to

$\overrightarrow{M} \in R^{t \times N}$

$\exists \overrightarrow{K}$: $the\ matrix\ of\ eigenvectors\ derived\ from\ \overrightarrow{M}\overrightarrow{M}^t$ $\quad \overrightarrow{K}^t\overrightarrow{K} = I$

$\overrightarrow{D}$: $the\ matrix\ of\ eigenvectors\ derived\ from\ \overrightarrow{M}^t\overrightarrow{M}$ $\quad \overrightarrow{D}^t\overrightarrow{D} = I$

$\overrightarrow{M} = \overrightarrow{K}\overrightarrow{S}\overrightarrow{D}^t$

$$\vec{M}^t \vec{M} : document-to-document\ matrix$$

$$= (\vec{K}\vec{S}\vec{D}^t)^t (\vec{K}\vec{S}\vec{D}^t)$$

$$= (\vec{D}\vec{S}^t\vec{K}^t)(\vec{K}\vec{S}\vec{D}^t)$$

$$= \vec{D}\vec{S}^2\vec{D}^t$$

$$\vec{M}\vec{M}^t : term-to-term\ matrix$$

$$= (\vec{K}\vec{S}\vec{D}^t)(\vec{K}\vec{S}\vec{D}^t)^t$$

$$= (\vec{K}\vec{S}\vec{D}^t)(\vec{D}\vec{S}^t\vec{K}^t)$$

$$= \vec{K}\vec{S}^2\vec{K}^t$$

對照A=QDQ^T

Q is matrix of eigenvectors of A

D is diagonal matrix of singular values

得到

$\vec{D}$: *the matrix of eigenvectors derived from* $\vec{M}^t \vec{M}$

$\vec{K}$: *the matrix of eigenvectors derived from* $\vec{M}\vec{M}^t$

$\vec{S}$: $r \times r$ *diagonal matrix of singular values, where* $r = \min(t, N)$

s < r (Concept space is reduced)

Consider only the s largest singular values of $\vec{S}$

$$\begin{pmatrix} \lambda_1 & & & & \\ & \lambda_2 & & 0 & \\ & & \cdot & & \\ & 0 & & \cdot & \\ & & & \cdot & \lambda_n \end{pmatrix}$$

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \geq 0$$

The resultant $\vec{M}_s$ matrix is the matrix of rank s which is closest to the original matrix M in the least square sense.

$$\overrightarrow{M}_s = \overrightarrow{K}_s \overrightarrow{S}_s \overrightarrow{D}_s^t$$

(s<<t, s<<N)

Ranking in LSI

- query: a pseudo-document in the original $\vec{M}$ term-document
  - query is modeled as the document with number 0
  - $\vec{M}_s^t \vec{M}_s$: the ranks of all documents w.r.t this query

Research Issues

- Library systems
  - Cognitive and behavioral issues oriented particularly at a better understanding of which criteria the users adopt to judge relevance
- Specialized retrieval systems
  - e.g., legal and business documents
  - how to retrieve all relevant documents without retrieving a large number of unrelated documents
- The Web
  - User does not know what he wants or has great difficulty in formulating his request
  - How the paradigm adopted for the user interface affects the ranking
  - The indexes maintained by various Web search engine are almost disjoint